Panel Presentation

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Session 6B

Easy Translation of TAR or TUR into Uncertainty

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Generic Uncertainty Expression

- **Standard uncertainty for a measurement result from a calibrated measurement device can be generally expressed as**

$$u(y) = \sqrt{u(D_n)^2 + u(C_n)^2}$$

where

$u(D_n)$ is the standard uncertainty of the measurement

$u(C_n)$ is the standard uncertainty of the calibration

Showing Next Level of Uncertainty

- **Expanding this formula to show the next level of uncertainty down the calibration chain gives**

$$u(y) = \sqrt{u(D_n)^2 + u(C_n)^2}$$

$$= \sqrt{u(D_n)^2 + \left[u(D_{n-1})^2 + u(C_{n-1})^2 \right]}$$

Assumptions

- Next, assume
  - TAR or TUR ≥4 at each level of calibration,
  - systems are in place that mitigate the effects of any potential sources of uncertainty not accounted for in the TAR or TUR being used, and
  - the numerator and denominator of the TAR or TUR are approximately known multiples of the associated standard uncertainties
- Example based on ANSI/NCSL Z540.3 TUR

$$TUR_{Z540.3} = \frac{\text{Upper Device Spec.} - \text{Lower Device Spec.}}{\text{Upper 95\% Cal. Unc.} - \text{Lower 95\% Cal. Unc.}}$$

$$\approx \frac{6 \cdot u(D_n)}{4 \cdot u(C_n)} = \frac{6}{4} \cdot \frac{u(D_n)}{u(C_n)} \equiv r_k \frac{u(D_n)}{u(C_n)}$$

use of 6/4 assumes approximate normality

use of r_k keeps results general

Relating Uncertainties at Different Levels

$$TUR \geq 4 \quad \Rightarrow \quad r_k \frac{u(D_n)}{u(C_n)} = r_k \frac{u(D_n)}{\sqrt{\left[u(D_{n-1})^2 + u(C_{n-1})^2\right]}} \geq 4$$

$$\Rightarrow \quad \frac{r_k^2}{16} u(D_n)^2 \geq \left[u(D_{n-1})^2 + u(C_{n-1})^2\right]$$

$$\Rightarrow \quad \frac{r_k^2}{16} u(D_n)^2 - u(D_{n-1})^2 - u(C_{n-1})^2 \geq 0$$

$$\Rightarrow \quad \frac{r_k^2}{16} u(D_n)^2 - u(D_{n-1})^2 \geq 0$$

Put These Expressions Together …

$$u(y) = \sqrt{u(D_n)^2 + \left[u(D_{n-1})^2 + u(C_{n-1})^2\right]}$$

$$\leq \sqrt{u(D_n)^2 + \left\{\frac{r_k^2}{16}u(D_n)^2 - u(D_{n-1})^2\right\} + \left[u(D_{n-1})^2 + u(C_{n-1})^2\right]}$$

$$= \sqrt{u(D_n)^2 + \frac{r_k^2}{16}u(D_n)^2 + u(C_{n-1})^2}$$

… and Just Carry On …

$$u(y) \le \sqrt{u(D_n)^2 + \frac{r_k^2}{16} u(D_n)^2 + u(C_{n-1})^2}$$

$$= \sqrt{u(D_n)^2 + \frac{r_k^2}{16} u(D_n)^2 + \left[u(D_{n-2})^2 + u(C_{n-2})^2\right]}$$

$$\le \sqrt{u(D_n)^2 + \frac{r_k^2}{16} u(D_n)^2 + \left\{\left(\frac{r_k^2}{16}\right)^2 u(D_n)^2 - u(D_{n-2})^2\right\} + \left[u(D_{n-2})^2 + u(C_{n-2})^2\right]}$$

$$= \sqrt{u(D_n)^2 + \frac{r_k^2}{16} u(D_n)^2 + \left(\frac{r_k^2}{16}\right)^2 u(D_n)^2 + u(C_{n-2})^2}$$

… Until You End Up Here!

$$u(y) \le \sqrt{\sum_{i=0}^{n-1} \left(\frac{r_k^2}{16}\right)^i u(D_n)^2}$$

$$\le \sqrt{\sum_{i=0}^{\infty} \left(\frac{r_k^2}{16}\right)^i u(D_n)^2}$$

$$= \sqrt{\frac{16}{16 - r_k^2}}\, u(D_n) \text{ if } \frac{r_k^2}{16} < 1$$

Now we can just use

$$u(y) = \frac{4(\text{Device Accuracy})}{\sqrt{16 - r_k^2}\sqrt{3}}$$

for legacy systems with TAR or TUR ≥4.

No further uncertainty analysis required!

Similar results appear to hold for systems based on EOPR as well.